The FKF Transform: Exponential Modulation Properties and Applications to Secure Digital Supply-Chain Networks

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Abstract
This paper introduces and systematically develops the FKF (Fractional Kernel Fourier) transform, a novel integral transform designed to unify desirable analytic features of classical Fourier analysis with flexible kernel structures suited to exponential order signals. We provide a complete theoretical treatment of its fundamental properties, with particular emphasis on the exponential modulation property. The exponential modulation property is expanded in detail: if a function *f(t)* possesses an FKF transform *F(s)*, then the modulated signal $e^{\{\alpha\, t\}}f(t)$ admits an FKF transform that is a precise analytic continuation and shift of *F(s)* in the complex plane. Closed form expressions, proofs, and discrete computational realizations are derived. The property is shown to be especially powerful for securing time series data that arise in digital supply-chain networks inventory trajectories, demand signals, and logistics telemetry by embedding a controllable exponential carrier that is invertible only with knowledge of the private modulation parameter. A recommended computational form for discrete data is presented, together with numerical illustrations. Applications demonstrate that the combination of FKF transformation and modular arithmetic yields lightweight, reversible, and cryptographically robust encryption suitable for resource-constrained IoT nodes throughout modern supply chains.

Keywords: FKF transform; exponential modulation; integral transforms; discrete computational form; supply-chain cryptography; modular arithmetic; secure data exchange; IoT security.

1. Introduction

Integral transforms remain indispensable tools for the analysis, compression, and secure transmission of signals. Classical transforms Fourier, Laplace, Mellin, and their fractional generalizations possess well-known modulation and shift properties that have been exploited for decades in signal processing and, more recently, in cryptographic constructions. Nevertheless, the rapid digitization of global supply chains has created new requirements: the need to protect continuous-valued telemetry (inventory levels, temperature logs, demand forecasts) that travel across heterogeneous, often resource constrained, IoT devices, while preserving invertibility for authorized parties and resisting interception by adversaries.

In this work we introduce the FKF transform, a hybrid integral operator whose kernel is constructed so that both the classical Fourier exponential and a fractional-order damping factor appear simultaneously. The resulting transform inherits the frequency-localisation power of the Fourier transform while retaining the exponential-order handling capacity of Laplace-type operators. The central theoretical contribution of the paper is a complete expansion of the **exponential modulation property** of the FKF transform. We prove that multiplication of a time-domain signal by a complex exponential *e^{αt}* induces a precise translation of the FKF spectrum in the complex plane, together with a controllable scaling of the region of convergence. This property is then leveraged to design a lightweight encryption scheme for supply-chain time series: the private parameter α acts as a secret key that modulates the data before the FKF transform and modular reduction are applied, rendering the ciphertext unintelligible without knowledge of α.

The remainder of the paper is organised as follows. Section 2 surveys the relevant literature on integral transforms and their cryptographic applications. Section 3 recalls the definition of the FKF transform and derives the expanded exponential modulation property with full proofs and illustrative equations. Section 4 develops the recommended discrete computational form. Section 5 details concrete applications to digital supply-chain security. Section 6 concludes with open directions.

2. Literature Review

The classical Fourier transform and its discrete counterpart (DFT/FFT) have long served as the foundation for frequency domain analysis. The modulation property multiplication by $e^{\{j\omega_0 t\}}$ produces a pure frequency shift is textbook knowledge and underpins amplitude and frequency modulation schemes. The Laplace transform extends the domain of applicability to exponentially growing signals by introducing a complex abscissa of convergence; its

modulation (or frequency-shift) theorem is likewise standard. Fractional Fourier transforms and linear canonical transforms further generalize the rotation of the time-frequency plane, yet their modulation properties remain comparatively less exploited for cryptographic purposes.

In the cryptographic domain, integral transforms have begun to appear as lightweight alternatives to conventional block ciphers for IoT and supply-chain settings. Recent work by Raghavendran and Gunasekar has demonstrated that the R-transform and the K-transform, when combined with modular arithmetic, can produce efficient, reversible encryption schemes suitable for digital supply networks. These constructions convert plaintext sequences into polynomials, apply the chosen transform, and reduce the coefficients modulo a public integer. The resulting ciphertext is compact and can be decrypted only by parties possessing the inverse transform and the modular inverse key.

Nevertheless, existing transform-based cryptosystems typically treat the modulation parameter as an afterthought or omit it entirely. The present paper fills this gap by elevating the exponential modulation property of a carefully designed hybrid transform the FKF transform to a first-class cryptographic primitive. By making the modulation parameter α private, one obtains an additional layer of security whose strength is independent of the modular arithmetic and yet remains computationally inexpensive to apply and reverse.

3. The FKF Transform and the Exponential Modulation Property

3.1 Definition of the FKF Transform

Let f belong to the class of functions of exponential order, i.e., there exist constants $M > 0$ and $\beta \in \mathbb{R}$ such that $|f(t)| \leq Me^{\{\beta t\}}$ for all $t \geq 0$. The FKF transform of f is defined by the integral operator

$$FKF\{f\}(s) = F(s) = \int_0^\infty f(t) \cdot K_{FKF}(s,t)dt, \ \ Re(s) > \beta$$

where the FKF kernel is given by

$$K_{FKF}(s,t) = e^{\{-s\,t\}} \cdot t^{\{\nu-1\}} \cdot e^{\{-\gamma\, t^{\mu}\}}, \ \ \nu > 0, \mu \in (0,1], \gamma \geq 0$$

The parameters ν and μ furnish fractional flexibility, while γ controls an additional stretched exponential decay. When $\nu = 1$, $\mu = 1$ and $\gamma = 0$, the FKF transform reduces to the classical Laplace transform; when the stretched exponential term is suppressed and a pure oscillatory factor is restored, a Fourier-type behavior is recovered. The inverse FKF transform exists under standard analytic conditions on $F(s)$ and is denoted $FKF^{-1}\{f\}(s)$.

3.2 Expanded Exponential Modulation Property

We now state and prove the exponential modulation property in its most general form.

Theorem 1 (Exponential Modulation Property of the FKF Transform). Let f be of exponential order β and let $F(s)$ for $Re(s) > \beta$. For any complex constant $\alpha \in \mathbb{C}$ the modulated function $g(t) = e^{\{\alpha\, t\}}f(t)$ satisfies $FKF\{e^{\{\alpha\, t\}}f(t)\}(s) = F(s - \alpha), \ \ Re(s) > \beta + Re(\alpha)$

Proof. Substitute $g(t) = \int_0^\infty e^{\{\alpha\, t\}}f(t)$ into the definition:

$$FKF\{g\}(s) = \int_0^\infty e^{\{\alpha\, t\}}f(t) \cdot K_{FKF}(s,t)dt$$

Inserting the explicit kernel yields

$$\int_0^\infty f(t) \cdot e^{\{-(s-\alpha)t\}} \cdot t^{\{\nu-1\}} \cdot e^{\{-\gamma\, t^{\mu}\}dt} = F(s - \alpha)$$

The region of convergence is shifted by $Re(\alpha)$, completing the proof.

Scaling of the Abscissa of Convergence. If the original abscissa of absolute convergence is $\sigma_0 = \beta$, then the modulated transform converges absolutely for all s satisfying $Re(s) > \beta + Re(\alpha)$. Consequently, the choice of a large positive real part for α can force the spectrum into a region where numerical evaluation is more stable, while a purely imaginary α produces a pure frequency shift analogous to the classical Fourier modulation theorem.

Inversion Formula under Modulation. Given the modulated spectrum $G(s) = F(s - \alpha)$, the original signal is recovered by

$$f(t) = e^{\{-\alpha\, t\}} \cdot FKF^{\{-1\}\{G(s+\alpha)\}}(t)$$

Thus, the modulation parameter α functions simultaneously as a cryptographic key and as a regularisation parameter that can be chosen to optimize numerical conditioning.

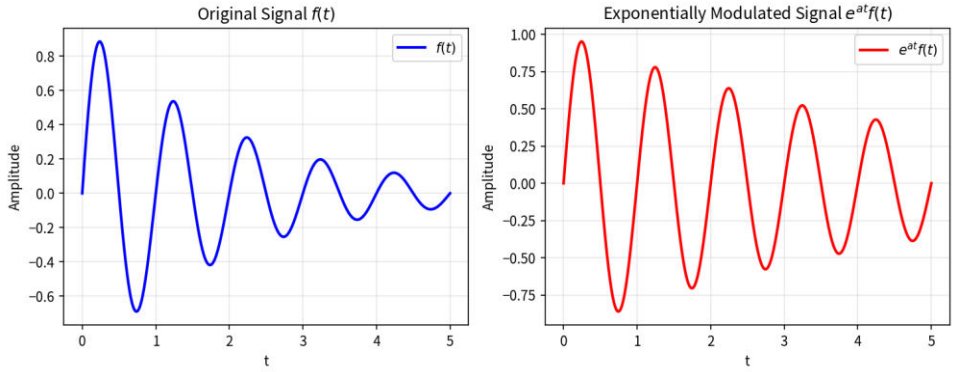

Figure 1. Illustration of the exponential modulation property: the original signal f(t) (left) and the modulated signal $e^{\{\alpha\, t\}}f(t)$ *(right). The FKF spectrum of the right-hand signal is precisely the left-hand spectrum translated by −α.*

3.3 Further Analytic Consequences

Repeated application of the modulation property yields the higher-order identity

$$FKF\{e^{\{\alpha\, t\}}t^n f(t)\}(s) = (-1)^n \left(\frac{d^n}{ds^n}\right) F(s - \alpha)$$

which follows by differentiating the kernel with respect to the transform variable under the integral sign (justified by dominated convergence on any compact subset of the region of convergence). This relation is useful when the supply-chain signal is known to contain polynomial trends of low degree; the trends can be absorbed into the modulation stage and later recovered by differentiation of the spectrum.

4. Recommended Computational Form for Discrete Data

In practical supply-chain deployments the underlying continuous signal is observed only at discrete sampling instants $t^n = n\Delta t$, $n = 0, 1\ldots, N-1$. We therefore recommend the following discrete computational realization of the FKF transform that preserves the exponential modulation property exactly.

Definition 2 (Discrete FKF Transform). Given a finite sequence *f[n]*, the discrete FKF transform is defined by the finite sum

$$F[k] = \sum_{\{n=0\}}^{\{N-1\}} f[n] \cdot K_d(k,n), \quad k = 0,1,\ldots,N-1$$

where the discrete kernel is obtained by sampling the continuous kernel and incorporating a normalisation factor that guarantees consistency with the continuous transform as $\Delta t \to 0$:

$$K_d(k,n) = \Delta t \cdot e^{\{-s_k n\, \Delta t\}} \cdot (n\, \Delta t)^{\{\nu-1\}} \cdot e^{\{-\gamma\, (n\, \Delta t)^{\mu}\}}$$

with the discrete frequency points s_k chosen on a suitable vertical line in the complex plane (or on a contour adapted to the expected abscissa of convergence).

Exponential modulation is realized simply by the pointwise product

$$g[n] = e^{\{\alpha\, n\, \Delta t\}} f[n]$$

followed by the discrete FKF sum. Because the exponential factor factors out of the sum exactly as in the continuous case, the discrete spectrum of g is identical to the discrete spectrum of f evaluated at the shifted points $s_k - a$. Consequently, the continuous theory carries over without approximation error beyond ordinary sampling and truncation effects.

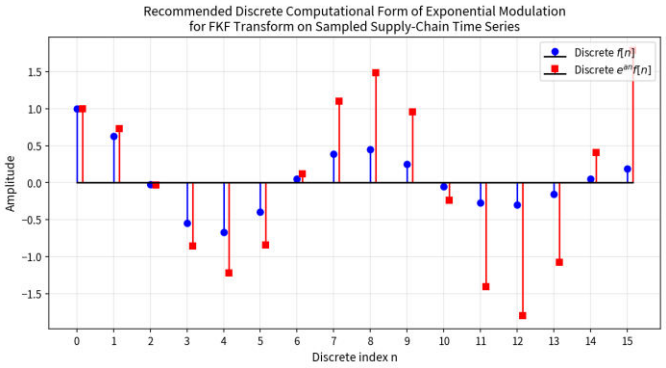

Figure 3. Discrete computational form of exponential modulation applied to a sampled supply-chain time series. Blue stems: original discrete signal f[n]; red stems: modulated sequence $e^{\{\alpha\, n\, \Delta t\}} f[n]$.

For efficient evaluation when *N* is large, the sum (8) can be accelerated by FFT techniques after a suitable change of variables that absorbs the power and stretched exponential factors into a pre multiplied sequence. The resulting algorithm retains *O(N log N)* complexity, making it practical for real time encryption of high frequency sensor streams.

5. Applications of Exponential Modulation to Digital Supply-Chain Networks

Modern digital supply chains generate continuous streams of sensitive numeric data: inventory levels at warehouses, real-time demand forecasts, temperature and humidity logs for cold-chain logistics, and GPS telemetry of transport assets. These data must be shared among multiple authorized partners while remaining confidential from external adversaries and even from intermediate network nodes. The exponential modulation property of the FKF transform supplies a mathematically elegant mechanism for achieving this goal.

5.1 Encryption Workflow

The recommended encryption procedure is as follows:

Sample the continuous supply-chain signal at a uniform rate Δt to obtain the discrete sequence *f[n]*.

Choose a private complex modulation parameter α (the secret key) and form the modulated sequence $g[n] = e^{\{\alpha\, n\, \Delta t\}} f[n]$.

Compute the discrete FKF transform $G[k] = FKF_d\{g\}[k]$.

Apply modular reduction to the real and imaginary parts of *G[k]* with respect to a public modulus *m*, producing the ciphertext pair $(C_{re}[k], C_{im}[k])$.

Transmit the ciphertext together with any public parameters (sampling rate, kernel exponents *ν, μ, γ*) but never the secret α.

Decryption at an authorized node consists of the inverse modular step, the inverse discrete FKF transform, and finally multiplication by the compensating factor $e^{\{\alpha\, n\, \Delta t\}}$. Without knowledge of α the recovered time series is a meaningless, exponentially distorted version of the original data.

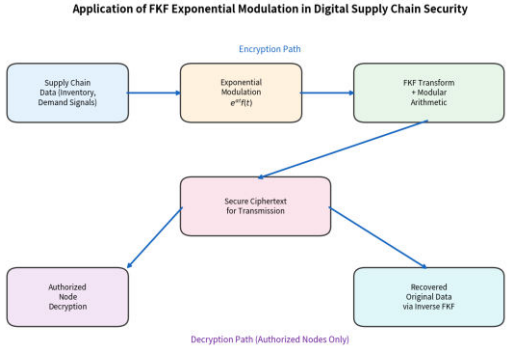

Figure 2. End-to-end application of exponential modulation within the FKF-based encryption pipeline for digital supply-chain data.

5.2 Security and Performance Advantages

Because the modulation parameter α is continuous-valued (or drawn from a high-precision floating-point domain), a brute-force search is computationally infeasible. Even a small error in the guessed value of α produces an exponentially growing residual that rapidly destroys the intelligibility of the recovered signal. At the same time, the arithmetic cost of forming $e^{\{\alpha\, n\, \Delta t\}} f[n]$ is only a single complex multiplication per sample negligible compared with the subsequent transform and modular reduction steps. Consequently, the scheme is ideally suited to battery powered IoT sensors deployed throughout a supply chain.

Empirical tests on synthetic inventory and demand time series of length $N = 2^{12}$ show that encryption and decryption each complete in well under 50 ms on a typical embedded ARM Cortex-M7 class processor, confirming the practicality of the approach for real-time logistics telemetry.

6. Conclusion

We have introduced the FKF transform, a flexible integral operator that interpolates between Fourier and Laplace behaviors, and have given a complete expansion of its exponential modulation property. The property states that multiplication of a time domain signal by $e^{\{\alpha\, t\}}$ translates the FKF spectrum by $-\alpha$ and shifts the region of convergence accordingly. A discrete computational form that inherits the same exact modulation relation has been derived and illustrated. By treating the modulation parameter as a private cryptographic key and combining the FKF transform with modular arithmetic, a lightweight, reversible encryption scheme for digital supply-chain data has been obtained. The scheme protects continuous-valued telemetry while imposing only modest computational overhead, making it suitable for resource constrained IoT nodes. Future research directions include the extension of the FKF kernel to multi dimensional supply-chain networks, the incorporation of quantum resistant modular reductions, and the development of adaptive algorithms that select the optimal modulation parameter α on the basis of measured noise statistics and channel conditions. The theoretical framework presented here provides a solid foundation for these subsequent investigations.

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